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ON THE GENERATION OF WAVES IN A PRESTRESSED LAYER

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The linearized theory of propagation of elastic waves /1,2/ is used as the starting point in the study of specific features of the excitation of a prestressed layer clamped rigidly along its lower edge, by a vibrating stamp. The medium is assumed to be compressible, initially isotropic and possessing an elastic potential of arbitrary form, and the vibrations of the stamp are assumed harmonic. Integral equations and systems of integral equations are constructed for the problems of a stamp of arbitrary and circular cross-section in the plane and of strip form, vibrating on the surface of a layer.

The problem of vibration of a stamp lying on the surface of a layer without friction is used to carry out a numerical analysis of the dispersion curves, and in the case of a strip-like stamp, the dependence of the contact pressure on the intensity of the initial state of stress of the medium is studied. The latter is assumed homogeneous, i.e. /1,2/

$$u_n^\circ = \delta_{in}(\lambda_i - 1)x_n, \quad n = 1, 2, 3$$
$$\lambda_i = \text{const}, \quad \lambda_1 = \lambda_2 \neq \lambda_3, \quad \sigma_{11}^{*\circ} = \sigma_{22}^{*\circ} \neq \sigma_{33}^{*\circ}$$

(u_n° are the components of the initial displacement of the points of the layer, λ_i are the relative elongations of the fibers, $\sigma_{ii}^{*\circ}$ are the components of the generalized initial stress tensor, and δ_{in} is the Kronecker delta).

1. Solution of the problem of excitation of the layer $0 \leqslant x_3 \leqslant h$, $|x_1|$, $|x_2| < \infty$ with the properties given above, by a surface load.

$$\mathbf{q}(x_1, x_2)e^{-i\omega t}, \quad \mathbf{q} = \{q_1, q_2, q_3\}, \qquad x_1, x_2 \in \Omega, \; u^2 = \alpha^2 + \beta^2 \tag{1.1}$$

(α, β are the parameters of the Fourier transformation with respect to the variables x_1 and x_2, and Ω is the region outside which the load is absent) can be written, using the principle of limiting absorption, in the form

$$u_k(x_1, x_2, 0) = 0, \; k = 1, 2, 3, \qquad u_j(x_1, x_2, x_3, t) = U_j(x_1, x_2, x_3)e^{-i\omega t}, \; j = 1, 2, 3 \tag{1.2}$$

$$U_j(x_1, x_2, x_3) = \frac{1}{4\pi^2}\sum_{n=1}^{3}\iint_{\Omega} q_n(\xi, \eta)k_{jn}(x_3, \xi - x_1, \eta - x_2)\,d\xi\,d\eta \tag{1.3}$$

$$k_{jn}(x_3, s, t) = \int_{\Gamma_1}\int_{\Gamma_2}\Delta_j^n e^{i[\alpha s + \beta t]}\,d\alpha\,d\beta$$

$$\Delta_1^n = \Delta_{1n}(\operatorname{sh}\sigma_1 x_3 + k\operatorname{sh}\sigma_2 x_3) + \Delta_{2n}(\operatorname{ch}\sigma_1 x_3 - \operatorname{ch}\sigma_2 x_3) + \Delta_{3n}\operatorname{sh}\sigma_3 x_3 \tag{1.4}$$

$$\Delta_2^n = [\Delta_{1n}(\operatorname{sh}\sigma_1 x_3 + k\operatorname{sh}\sigma_2 x_3) + \Delta_{2n}(\operatorname{ch}\sigma_1 x_3 - \operatorname{ch}\sigma_2 x_3)] \times \beta\alpha^{-1} - \alpha\beta^{-1}\Delta_{3n}\operatorname{sh}\sigma_3 x_3$$

$$\Delta_3^n = i\alpha^{-1}[\Delta_{1n}(f_1\operatorname{ch}\sigma_1 x_3 + kf_2\operatorname{ch}\sigma_2 x_3) + \Delta_{2n} \times (f_1\operatorname{sh}\sigma_1 x_3 - f_2\operatorname{sh}\sigma_2 x_3)]$$

$$\Delta_{11} = -l_3 T_1 T_5, \;\; \Delta_{12} = \alpha\beta^{-1}T_1T_5, \;\; \Delta_{13} = i\alpha(l_3 - 1)T_2T_5$$

$$\Delta_{21} = l_3 T_3 T_5, \;\; \Delta_{22} = \alpha\beta^{-1}T_3T_5, \;\; \Delta_{23} = -i\alpha(l_3 - 1)T_4T_5$$

$$\Delta_{31} = T_1T_4 - T_2T_3, \;\; \Delta_{32} = -\alpha\beta^{-1}\Delta_{31}, \;\; \Delta_{33} = 0, \;\; \Delta = (1 - l_3) \times T_5\Delta_{31}$$

$$T_1 = m_1\operatorname{ch}\sigma_1 h - m_2\operatorname{ch}\sigma_2 h, \quad T_2 = \frac{l_1}{\sigma_1}\operatorname{sh}\sigma_1 h - \frac{l_2}{\sigma_2}\operatorname{sh}\sigma_2 h$$

$$T_3 = m_1\operatorname{sh}\sigma_1 h - km_2\operatorname{sh}\sigma_2 h, \quad T_4 = \frac{l_1}{\sigma_1}\operatorname{ch}\sigma_1 h + k\frac{l_2}{\sigma_2}\operatorname{ch}\sigma_2 h, \qquad T_5 = A_3\sigma_3\operatorname{ch}\sigma_3 h$$

$$l_k = A_3\sigma_k^2 + \lambda_1\lambda_3\mu_{13}d_k, \quad k = 1, 2, \quad d_k = -\frac{A_3}{A_5}\sigma_k^2 + \frac{S_1}{A_5} \tag{1.5}$$

$$m_k = \lambda_1\lambda_3 a_{31}u^2 - A_6 d_k, \quad l_3 = -\frac{\alpha^2}{\beta^2}, \quad k = \frac{d_1\sigma_2}{d_2\sigma_1}, \quad f_k = -i\frac{\alpha d_k}{\sigma_k}$$

$$\sigma_{1,2} = \left(\frac{D_2 \pm \Sigma^{1/2}}{2D_1}\right)^{1/2}, \quad \sigma_3 = \left(\frac{A_2u^2 - \rho\omega^2}{A_3}\right)^{1/2} \tag{1.6}$$

$$D_1 = A_3A_6, \;\; D_2 = A_3S_2 + A_6S_1 - A_5^2u^2, \;\; D_3 = S_1S_2$$

$$S_1 = A_1u^2 - \rho\omega^2, \;\; S_2 = A_7u^2 - \rho\omega^2, \;\; \Sigma = D_2^2 - 4D_1D_3$$

$$A_1 = a_{11}\lambda_1^2 + \sigma_{11}^{*\circ}, \;\; A_2 = \mu_{12}\lambda_1^2 + \sigma_{22}^{*\circ}, \;\; A_3 = \mu_{13}\lambda_1^2 + \sigma_{33}^{*\circ} \tag{1.7}$$

$$A_4 = \lambda_1\lambda_2(a_{12} - \mu_{12}), \;\; A_5 = \lambda_1\lambda_3(a_{13} + \mu_{13}), \qquad A_6 = a_{33}\lambda_3^2 + \sigma_{33}^{*\circ}, \;\; A_7 = \mu_{13}\lambda_3^2 + \sigma_{11}^{*\circ}$$

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Here ρ is the density of the material of the medium, a_{ik} and μ_{ik} are the coefficients characterizing the stress-strain relationships and determined with the help of elastic potential /1, 2/. Their concrete form for the case of Murnaghan potential considered here will be given below. The contours Γ_k (with the exception of Γ_3) lie on the real axis and deviate from it only to pass above or below the negative and positive singularities of the integrand function. Their choice is dictated by the principle of limiting absorption /3,4,5/. The right hand side of (1.3) determines the displacement of any point of the layer in which the initial stresses are caused by a load distributed in Ω.

2. Setting in (1.3) $x_3 = h$, we obtain the displacement of the layer surface determined by the relations ($\mathbf{q}(\xi, \eta) = \{q_1, q_2, q_3\}$ in the stress vector and $\mathbf{u}_0(x_1, x_2) = \{u_1, u_2, u_3\}$ is the layer surface displacement vector)

$$u_0(x_1, x_2) = \frac{1}{4\pi^2} \iint_\Omega q(\xi, \eta) k(\xi - x_1, \eta - x_2)\, d\xi, d\eta \tag{2.1}$$

$$\mathbf{k} = \begin{Vmatrix} \alpha^2 M + \beta^2 N & \alpha\beta(M - N) & -i\alpha S \\ \alpha\beta(M - N) & \alpha^2 N + \beta^2 M & -i\beta S \\ i\alpha S & i\beta S & R \end{Vmatrix} \tag{2.2}$$

$$M(u) = \sigma_1\sigma_2(m_2 - m_1)(d_2\,\sigma_2\, \mathrm{ch}\, \sigma_1 h\, \mathrm{sh}\, \sigma_2 h - d_2\,\sigma_1 \mathrm{ch}\, \sigma_2 h\, \mathrm{sh}\, \sigma_1 h) / (u^2 \Delta(u)) \tag{2.3}$$

$$N(u) = \mathrm{th}\, \sigma_3 h / (u^2 A_3\, \sigma_3)$$

$$S(u) = [\sigma_1\sigma_2(l_1 d_2 + l_2 d_1)(1 - \mathrm{ch}\, \sigma_1 h\, \mathrm{ch}\, \sigma_2 h) + \mathrm{sh}\, \sigma_1 h\, \mathrm{sh}\, \sigma_2 h\, (l_1 d_1\, \sigma_1^2 + l_2 d_2\, \sigma_2^2)] / \Delta(u)$$

$$R(u) = (d_1 l_2 - d_2 l_1)(d_1\,\sigma_2\, \mathrm{sh}\, \sigma_1 h\, \mathrm{ch}\sigma_2 h - d_2\sigma_1\, \mathrm{sh}\, \sigma_2 h \times \mathrm{ch}\, \sigma_1 h) / \Delta(u)$$

$$\Delta(u) = \sigma_1\sigma_2(l_1 m_1 d_2 + l_2 m_2 d_1)(1 - \mathrm{ch}\, \sigma_1 h\, \mathrm{ch}\, \sigma_2 h) + \mathrm{sh}\, \sigma_1 h\, \mathrm{sh}\, \sigma_2 h\, (l_1 m_2 d_1\, \sigma_2^2 + l_2 m_1 d_2\, \sigma_1^2)$$

In the case of the problem of a stamp acting on the surface of a layer, the relation (2.1) represents a system of integral equations in terms of the contact stress vector. The functions appearing in (2.2) are analytic in the complex plane, real on the real axis and possess, on this axis, an enumerable set of zeros and poles the number of which depends on the frequency ω. It is clear that when $u \to \infty$,

$$S(u) = c_3 / u^2 + O(u^{-4}),\ M(u) = c_1 / u^4 + O(u^{-6})$$
$$R(u) = c_4 / u + O(u^{-3}),\ N(u) = c_2 / u^3 + O(u^{-5})$$

Using the relations (1.6), we can obtain the expressions for the coefficients c_k, which are too bulky and hence not given here.

3. For the practically important case of vibration of a stamp of circular cross section, we obtain the following relations on the layer surface by passing in (2.1)—(2.3) to a cylindrical coordinate system (u_r, u_z and u_φ are the radial, axial and torsional displacements of the layer surface, and q_r, q_z and q_φ are the corresponding stresses under the stamp of radius a):

For the axisymmetric oscillations we have

$$u_r = \int_0^a q_r(\rho) k_{11}(r, \rho)\rho\, d\rho + \int_0^a q_z(\rho) k_{12}(r, \rho)\rho\, d\rho, \quad u_z = \int_0^a q_r(\rho) k_{21}(r, \rho)\rho\, d\rho + \int_0^a q_z(\rho) k_{22}(r, \rho)\rho\, d\rho \tag{3.1}$$

and for the skew symmetric oscillations we have

$$u_\varphi = \int_0^a q_\varphi(\rho) k_{33}(r, \rho)\rho\, d\rho \tag{3.2}$$

where

$$k_{ii}(r, \rho) = \int_{\Gamma_3} K_{ii}(u) J_1(ur) J_1(u\rho) u\, du, \quad i = 1, 3, \qquad k_{12}(r, \rho) = \int_{\Gamma_3} K_{12}(u) J_1(ur) J_0(u\rho) u\, du$$

$$k_{21}(r, \rho) = \int_{\Gamma_3} K_{21}(u) J_0(ur) J_1(u\rho) u\, du, \qquad k_{22}(r, \rho) = \int_{\Gamma_3} K_{22}(u) J_0(ur) J_0(u\rho) u\, du$$

$$K_{11}(u) = u^2 M(u),\ K_{12}(u) = -K_{21}(u) = uS(u), \qquad K_{22}(u) = R(u),\ K_{33}(u) = u^2 N(u)$$

The functions $M(u)$, $R(u)$, $S(u)$ and $N(u)$ are defined by the formulas (2.3). The contour Γ_3 lies on the positive part of the real axis, deviating from it to pass the singularities of the integrand function from below /5,6/.

4. Problem of vibration of a strip stamp of width $2a$, directed along the axis x_2, on the layer surface, reduces in the case of plane oscillations to the following systems of equations:

$$\mathbf{u}(x_1)=\frac{1}{2\pi}\int_{-a}^{a}\mathbf{q}(\xi)k(x_1-\xi)\,d\xi,\quad \mathbf{u}(x_1)=\{u_1,u_3\},\quad \mathbf{q}(x_1)=\{q_1,q_3\} \tag{4.1}$$

and in the case of antiplane oscillations to the integral equation

$$u_2(x_1)=\frac{1}{2\pi}\int_{-a}^{a}q_2(\xi)k_{22}(x_1-\xi)\,d\xi,\quad |x_1|\leqslant a \tag{4.2}$$

$$\mathbf{k}(t)=\int_{\Gamma_1}\mathbf{K}(\alpha)e^{i\alpha t}\,d\alpha,\quad \mathbf{K}(\alpha)=\begin{Vmatrix}K_{11}(\alpha) & iK_{13}(\alpha)\\ -iK_{13}(\alpha) & K_{33}(\alpha)\end{Vmatrix}$$

$$k_{22}(t)=\int_{\Gamma_1}K_{22}(\alpha)e^{i\alpha t}\,d\alpha$$

$$K_{11}(\alpha)=\alpha^2M(\alpha),\quad K_{13}(\alpha)=\alpha S(\alpha),\ K_{33}(\alpha)=R(\alpha),\qquad K_{22}(\alpha)=\alpha^2N(\alpha) \tag{4.3}$$

The functions $M(\alpha)$, $S(\alpha)$, $R(\alpha)$, $N(\alpha)$ are given by the formulas (2.3), and α is the parameter of the one-dimensional Fourier transformation with respect to the variable x_1.

5. Integral equations and their systems similar to those discussed on Sects.1—4, were dealt with in /3,7—11/. Generally, it is not possible to verify whether the conditions of unique solvability given in these papers hold. Additional numerical analysis is required in every particular case (a material with a given state law and physico-mechanical properties, a definite character and intensity of the initial stresses).

The present paper is concerned with the specific features of the process of wave generation in a prestressed layer, by a vibrating stamp. This leads to the analysis of solutions of the integral equations of the corresponding problems constructed here. Some of the relationships can be established by studying the properties of the symbols of the kernels of integral equations, and in particular their dispersion curves.

As an example, we consider a problem of a stamp vibrating in the direction normal to the layer surface, in the absence of friction between the stamp and the layer. We assume that the medium is hypereleastic with the Murnaghan potential, and the initial state of stress is described by the condition

$$\sigma_{11}^{*\circ}=\sigma_{22}^{*\circ}=s_1,\quad \sigma_{33}^{*\circ}=s_2$$

The problem reduces to that of solving the integral equation

$$u_3(x_1)=\frac{1}{2\pi}\int_{-a}^{a}q_3(\xi)k_{33}(x_1-\xi)\,d\xi,\qquad k_{33}(t)=\int_{\Gamma}K_{33}(u)e^{iut}\,du \tag{5.1}$$

The symbol $K_{33}(u)$ is defined in (4.3). The coefficients a_{ik} and μ_{ik} appearing in the expressions (1.5)—(1.7) have, under the assumptions made, the form

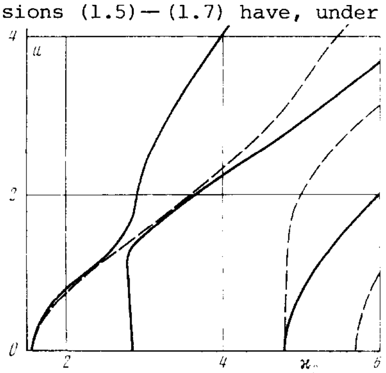

Fig.1

$$a_{ii}=\lambda+2\mu+k_0(a_{ii}^0s_1+a_{ii}^1s_2),\quad i=1,2,3$$
$$a_{1i}=\lambda+k_0(a_{1i}^0s_1+a_{1i}^1s_2),\quad i=2,3$$
$$\mu_{1i}=\mu+k_0(m_{1i}^0s_1+m_{1i}^1s_2),\quad i=2,3$$
$$a_{11}^0=4a+2(4+\gamma)b+(2+\gamma)c,\quad a_{11}^1=2a+2(1-\gamma)\times b-\gamma c$$
$$a_{33}^0=8a+8(1-\gamma)b-4\gamma c,\quad a_{33}^1=2a+(6+4\gamma)b+2(1+\gamma)c$$
$$a_{12}^0=2a+(2+\gamma)b,\quad a_{12}^1=a-\gamma b$$
$$a_{13}^0=4a+(2-\gamma)b,\quad a_{13}^1=2a+2(3+2\gamma)b+2\times(1+\gamma)c$$
$$m_{12}^0=2b+(2+\gamma)c/2,\quad m_{12}^1=b-\gamma c/2$$
$$m_{13}^0=2b+(2-\gamma)c/4,\quad m_{13}^1=b+(2+\gamma)c/4$$
$$\lambda_1^2=1+k_0[(2+\gamma)s_1-\gamma s_2],\quad \lambda_3^2=1+k_0[-2\gamma s_1+2\times(1+\gamma)s_2]$$
$$\gamma=\lambda/\mu,\quad k_0=(3\lambda+2\mu)^{-1}$$

(μ and λ are the Lamé parameters and a, b, c are third order constants appearing in the expression for the Murnaghan potential /1,2/).

The problem of distribution of zeros and poles of the function $K_{33}(u)$ (4.3) is important in proving the unique solvability of the equation. Solving the equations $K_{33}(\zeta_k,\varkappa_2)=0$ and $K_{33}(z_k,\varkappa_2)=0$ $(\varkappa_2=a\omega(\rho/\mu)^{1/2})$ for the zeros ζ_k and poles z_k, we obtain the functions $\zeta_k(\varkappa_2)$ and $z_k(\varkappa_2)$ (k is the consecutive numbers of the zero or pole). Fig.1 depicts the graphs of these functions for $s_1=s_2=0$, with the poles represented by solid lines and the zeros by dashed lines. Such a distribution of the curves is characteristic also for the values of $\sigma_{33}^{*\circ}$ different from zero and this implies, according to /6/, the existence of a solution of (5.1) unique in L_α $(\alpha>1)$ at all frequencies, for the values of s_2 considered here $(s_1=0)$.

The computations were carried out for steel 09 G2S /2/ for the following values of the elastic constants:

$$\lambda = 9.26\cdot 10^{11}\ \text{N/m}^2;\quad \mu = 7.75\cdot 10^{10}\ \text{N/m}^2$$
$$a = -3.19\cdot 10^{11}\ \text{N/m}^2;\quad b = -3.03\cdot 10^{11}\ \text{N/m}^2,\quad c = -0.784\cdot 10^{11}\ \text{N/m}^2$$

Figure 2 shows how s_2 influences the dependence of $\eta_k = (z_k/z_{0k} - 1)\cdot 10^3$ on $\varkappa_2$, where z_{0k} denote the poles of $K_{33}(u)$ (4.3) at $s_1 = s_2 = 0$ and z_k denote the poles in the case when the initial state of stress is not zero. The numbers 1,2 and 3 denote the curves obtained for the values of s_2 equal, respectively, to $2\cdot 10^{-4}\mu, 5\cdot 10^{-4}\mu, 8\cdot 10^{-4}\mu$ where μ is the shear modulus. The graphs show the ranges of the values of $\varkappa_2$ over which s_2 exert a "weak" and a "strong" influence on $z_k(\varkappa_2)$. Comparing Fig.2 with Fig.1 we see, that the ranges of strong influence correspond to the neighborhoods of the points of inflection of the curves $z_k(\varkappa_2)$.

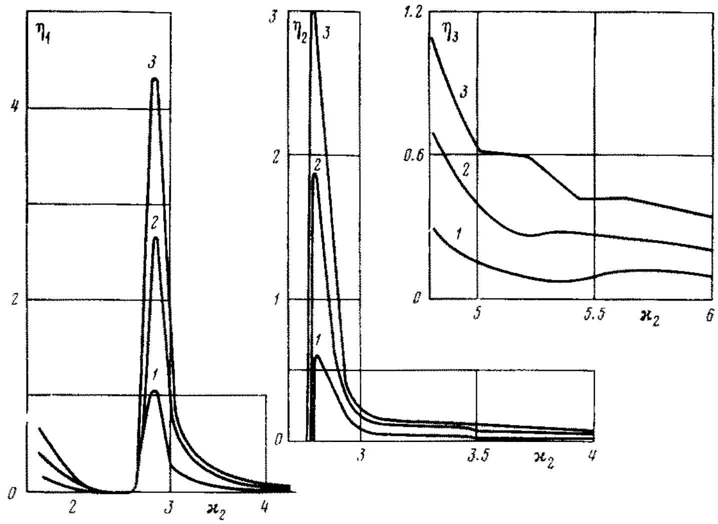

Fig.2

The range of variation of s_2 is made somewhat wider compared with that adopted in the literature /1,2/, in order to represent more accurately the influence of the intensity of the initial state of stress on the wave generating process under the stamp, as well as outside it. The behavior of the free surface outside the stamp can be described by the functions /10/ (S_k are numerical coefficients and B is the approximation parameter)

$$\varphi^{\pm}(x) = \varphi(\pm x - a),\quad \pm x - a \geqslant 1\,,\qquad \varphi(t) = \sum_{k=1}^{m} S_k e^{iz_k t} + O(e^{-Bt}) \tag{5.2}$$

From (5.2) it follows that in the neighborhood of the point of inflection of the curves $z_k(\varkappa_2)$ small changes in the value of $\sigma_{33}^{*\circ}$ may lead to arbitrarily sharp changes in the wave pattern at the layer surface.

Knowing the distribution of zeros and poles of $K_{33}(u)$ (4.3), we can construct the approximating function /3,7—13/

$$K^*(u) = c_1(u^2 + B^2)^{-1/2}\prod_{k=1}^{n}(u^2 - \zeta_k^2)(u^2 - z_k^2)^{-1} \tag{5.3}$$

The form of the solution in the case of $u_3(x) = \exp i\eta x$ and of the approximating function in various forms, are given in /3,7—13/.

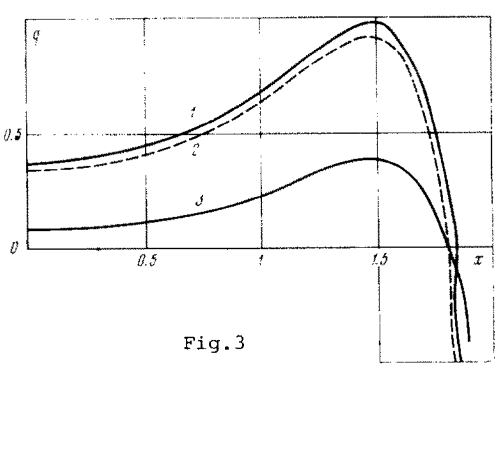

Fig.3

Figure 3 depicts the graphs of the function $q = \operatorname{Re} q_3\cdot\mu^{-1}$ constructed with the help of a digital computer for the case of a vibrating flat stamp for $\eta = 0$, $a = 2$, $\varkappa_2 = 2.79$ and for the values $\sigma_{33}^{*\circ} = 5\cdot 10^{-4}\mu, 10^{-3}\mu, 5\cdot 10^{-3}\mu$ (curves 1—3 respectively).

We note that the graphs in the Figs.1 and 2 enable use to establish the presence of a unique solution of the axisymmetric problem /4/ of vibration of a stamp of circular cross-section on the surface of a prestressed layer, and to clarify the influence of the magnitude of the initial stress on the wave formation outside the stamp. The method developed in /7,12,13/ can be used to compute the constact stresses under the stamp and to reveal the influence of the intensity of the prestressed state on their distribution.

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